Set-up for plotting evolutionary game dynamics in an infinite, well-mixed population
Uses replicator-mutator dynamics

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After running this notebook, run the EvoGameTutorialPlotter notebook.

```
Clear["`*"]

n = 3

3

f = {
   {b - c, -c, b},
   {b, 0, 0},
   {0, b, b - c}
  };

b = 3;
c = 1;

f // MatrixForm
```

$$\begin{pmatrix} 2 & -1 & 3 \\ 3 & 0 & 0 \\ 0 & 3 & 2 \end{pmatrix}$$

```
xvect =
 Table[ (Subscript[x, i])[t], {i, 1, n}]
(* Pop size vector *)
```

$\{x_1[t], x_2[t], x_3[t]\}$

```
xvectNoT =
 Table[ (Subscript[x, i]), {i, 1, n}]
```

$\{x_1, x_2, x_3\}$

```
xassign = Table[xvect[[j]] → (1 - xvect[[1]]) / (n - 1), {j, 2, n}]
```

$$\left\{x_2[t] \to \frac{1}{2}(1 - x_1[t]),\ x_3[t] \to \frac{1}{2}(1 - x_1[t])\right\}$$

```
xeqns = Table[ (xvect[[j]] /. {t → 0}) == (1 - (xvect[[1]] /. {t → 0})) / (n - 1), {j, 2, n}]
(* Equations used for init conditions in diff eq *)
```

$$\left\{x_2[0] == \frac{1}{2}(1 - x_1[0]),\ x_3[0] == \frac{1}{2}(1 - x_1[0])\right\}$$

```
q = IdentityMatrix[n]; (* Initialize mutation matrix *)

Do[
 Do[
  q[[i, j]] = If[i == j, 1 - u, u / (n - 1)],
   {j, 1, n}],
   {i, 1, n}
] (* Defines mutation matrix *)
```

```
q // MatrixForm
```

$$\begin{pmatrix} 1-u & \frac{u}{2} & \frac{u}{2} \\ \frac{u}{2} & 1-u & \frac{u}{2} \\ \frac{u}{2} & \frac{u}{2} & 1-u \end{pmatrix}$$

```
phi = Function[{x}, Simplify[(x.(f.x))]];
(* Average fitness *)
```

```
phi[{xc, xd, 1 - xc - xd}]
```

$$2 - xc + xc^2 - xd - xd^2$$

```
xdot = Function[{x, i},
   (Sum[
      x[[j]] * ( (f[[j]].x)) * q[[j, i]]
      , {j, 1, n}
     ]
     - phi[x] * x[[i]])
  ] ;
(* System follows replicator-mutator dynamics *)
```

```
Simplify[xdot[{xc, xd, 1 - xc - xd}, 1] /. u → 0]
```

$$xc\left(1 - xc^2 - 3\,xd + xd^2\right)$$

```
Simplify[xdot[{xc, xd, 1 - xc - xd}, 2] /. u → 0]
```

$$xd\left(-2 + 4\,xc - xc^2 + xd + xd^2\right)$$

```
soln[params_] :=
  NDSolve[
   Flatten[
    {
     ((Table[
         D[ (xvect[[i]]), t] ==
          xdot[xvect, i],
         {i, 1, n}
        ]) /. params),

     Table[
      (xvect /. {t → 0})[[i]] == (xinit /. params)[[i]],
      {i, 1, n}
     ]
     (* Initial conditions *)
    }
   ],

   xvectNoT,

   {t, 0, 1000},
   MaxSteps → 10^7
  ];
```

```
soln[{u → 0, xinit → {0.1, 0.1, 0.8}}]
```

NDSolve::ndsz :
At t == 289.29543517871684`, step size is effectively zero; singularity or stiff system suspected. ≫

```
{{x1 → InterpolatingFunction[{{0., 289.295}}, <>],
  x2 → InterpolatingFunction[{{0., 289.295}}, <>],
  x3 → InterpolatingFunction[{{0., 289.295}}, <>]}}
```

```
mysoln = Evaluate[
  xvect /.
   soln[{u → 0.0, xinit → {0.1, 0.1, 0.8}}]
 ]
```

NDSolve::ndsz :
At t == 289.2954355366268`, step size is effectively zero; singularity or stiff system suspected. ≫

```
{{InterpolatingFunction[{{0., 289.295}}, <>][t],
  InterpolatingFunction[{{0., 289.295}}, <>][t],
  InterpolatingFunction[{{0., 289.295}}, <>][t]}}
```

```
Table[mysoln /. t → t1, {t1, 0, 100, 1}]
```

```
{{{0.1, 0.1, 0.8}}, {{0.226875, 0.0257438, 0.747381}}, {{0.512665, 0.0128722, 0.474462}},
 {{0.830265, 0.0169206, 0.152814}}, {{0.932678, 0.0380891, 0.0292325}},
 {{0.899951, 0.0944292, 0.00561984}}, {{0.77892, 0.219459, 0.00162124}},
 {{0.566879, 0.432034, 0.00108689}}, {{0.326092, 0.671551, 0.00235679}},
 {{0.15257, 0.832885, 0.0145447}}, {{0.0667772, 0.791947, 0.141275}},
 {{0.0449763, 0.337556, 0.617468}}, {{0.0749981, 0.0701539, 0.854848}},
 {{0.18086, 0.0156223, 0.803517}}, {{0.436039, 0.00629283, 0.557668}},
 {{0.78717, 0.00683155, 0.205998}}, {{0.945825, 0.0147001, 0.0394749}},
 {{0.956071, 0.0375674, 0.00636177}}, {{0.903471, 0.0953429, 0.0011858}},
 {{0.777267, 0.22239, 0.000343022}}, {{0.562633, 0.437132, 0.000234151}},
 {{0.321459, 0.67802, 0.000521516}}, {{0.148735, 0.84792, 0.00334473}},
 {{0.061445, 0.899787, 0.0387685}}, {{0.0287474, 0.615802, 0.35545}},
 {{0.031022, 0.153501, 0.815477}}, {{0.0680758, 0.0269606, 0.904964}},
 {{0.175414, 0.00573479, 0.818852}}, {{0.431915, 0.00225654, 0.565829}},
 {{0.789703, 0.00243093, 0.207866}}, {{0.956107, 0.00527014, 0.0386226}},
 {{0.980543, 0.0137075, 0.00574964}}, {{0.962904, 0.0362156, 0.000880479}},
 {{0.907239, 0.0925994, 0.00016131}}, {{0.782834, 0.21712, 0.0000456831}},
 {{0.570168, 0.429801, 0.0000301743}}, {{0.327983, 0.671952, 0.0000648515}},
 {{0.152257, 0.847336, 0.000407364}}, {{0.0621076, 0.932993, 0.00489891}},
 {{0.0244638, 0.902662, 0.0728746}}, {{0.0130219, 0.463508, 0.52347}},
 {{0.0185161, 0.0903996, 0.891084}}, {{0.0445268, 0.0143446, 0.941129}},
 {{0.117892, 0.00262472, 0.879484}}, {{0.30589, 0.000753942, 0.693356}},
 {{0.656946, 0.000534897, 0.342519}}, {{0.920207, 0.000952536, 0.078841}},
 {{0.98564, 0.00240146, 0.0119583}}, {{0.991889, 0.00643298, 0.00167782}},
 {{0.98249, 0.0172699, 0.000240411}}, {{0.954381, 0.0455817, 0.0000376703}},
 {{0.885092, 0.1149, 7.43339×10^-6}}, {{0.739165, 0.260832, 2.47656×10^-6}},
 {{0.510411, 0.489587, 2.1349×10^-6}}, {{0.27721, 0.722783, 6.1144×10^-6}},
 {{0.12365, 0.876303, 0.0000468665}}, {{0.0493576, 0.950016, 0.000626093}},
 {{0.0188123, 0.970603, 0.0105851}}, {{0.00746416, 0.832437, 0.160099}},
 {{0.00522801, 0.289247, 0.705525}}, {{0.00965051, 0.0472275, 0.943122}},
 {{0.0246383, 0.0069564, 0.968405}}, {{0.0662178, 0.00111515, 0.932667}},
 {{0.177244, 0.000234029, 0.822522}}, {{0.439525, 0.0000931973, 0.560382}},
 {{0.799165, 0.000102805, 0.200732}}, {{0.963552, 0.000226114, 0.0362216}},
 {{0.994253, 0.0005948, 0.00515176}}, {{0.997687, 0.00160796, 0.00070548}},
 {{0.995547, 0.00435617, 0.0000968902}}, {{0.988234, 0.0117523, 0.0000136135}},
 {{0.968685, 0.0313134, 2.03593×10^-6}}, {{0.919227, 0.0807725, 3.58016×10^-7}},
 {{0.807197, 0.192803, 9.27986×10^-8}}, {{0.606327, 0.393673, 5.25184×10^-8}},
 {{0.361675, 0.638325, 9.4116×10^-8}}, {{0.172487, 0.827512, 5.16263×10^-7}},
 {{0.0712199, 0.928774, 5.82211×10^-6}}, {{0.0274364, 0.972471, 0.0000928627}},
 {{0.0102773, 0.988017, 0.00170539}}, {{0.00384629, 0.964337, 0.0318167}},
 {{0.00169929, 0.638917, 0.359384}}, {{0.00183177, 0.144671, 0.853497}},
 {{0.0041192, 0.0212404, 0.97464}}, {{0.0108899, 0.00298322, 0.986127}},
 {{0.0294739, 0.000435424, 0.970091}}, {{0.0798496, 0.0000719519, 0.920078}},
 {{0.212741, 0.0000164481, 0.787243}}, {{0.509308, 7.81697×10^-6, 0.490684}},
 {{0.849276, 0.0000103279, 0.150713}}, {{0.974809, 0.000024142, 0.0251666}},
 {{0.996415, 0.0000641759, 0.003521}}, {{0.999347, 0.000173897, 0.00047896}},
 {{0.999463, 0.000472363, 0.0000649564}}, {{0.998708, 0.00128291, 8.82798×10^-6}},
 {{0.996519, 0.0034796, 1.20778×10^-6}}, {{0.990597, 0.00940235, 1.6839×10^-7}},
 {{0.974848, 0.0251518, 2.46974×10^-8}}, {{0.934463, 0.0655374, 4.13007×10^-9}},
 {{0.839882, 0.160118, 9.534×10^-10}}, {{0.658665, 0.341335, 4.34975×10^-10}}}
```