Problem Set: Simulating a branching process

Please show all work and represent your answers in simplest possible form. Instructions for using Python are on the course website.

Question #1 (100 points)

We are interested in the dynamics of a branching process that starts with a single cell at time $t = 0$. During an infinitesimal time interval Δt, any cell still alive can divide into 2 cells with probability $b\Delta t$ or die with probability $d\Delta t$.

1a) (20 points)

Derive the probability distribution for T, the time until next event in this process (event being birth or death) in terms of b, d and n, where n is the number of cells currently alive. What's the name of this distribution?

1b) (50 points)

Using the result from part 1a) write a Python simulation which returns the average number of cells, $N(t)$, in this process at time t, using parameter values $b = 1.1$ and $d = 1$. Please treat time continuously, not as a sequence of small, discrete steps.

(i) Write a function one_run(t) which simulates the process and returns the number of cells num_cells at time t.

(ii) Write a function simulation(t,num_runs) which calls the function one_run(t) num_runs number of times and returns the average number of cells at time t across multiple runs.

(iii) What do you get for $N(t)$, when $t = 0, 10, 20, 30, 40, 50$? Make sure that you average over enough runs, so that you get a stable result (meaning that when you run the program again, the result doesn't change much). Attach your program to your solutions.

1c) (15 points)

Plot the values you got for $N(t)$ as a function of t in Mathematica. Can you guess what the formula for $N(t)$ is? Plot your guess against the values you got from 1b).

1d) (15 points)

Using a relation between $N(t + \Delta t)$ and $N(t)$, derive the formula for $N(t)$, the average number of cells in this process at time t.